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V

CONTRIBUTION TO THE OPTICS OF THE MICROSCOPE

BY

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Introduction

A subject which has received as much attention as the optics of the microscope is not liable to offer much that is new except to the delver into the intricacies of its details.

That which follows is not concerned with matters of detail, but instead, is based on a re-examination of some of the fundamental facts from a somewhat different angle than those by which they have usually been approached. This has resulted in the finding of some relationships which appear to have been overlooked and in the perception of the practical importance of some well known relationships in the calculation of lens aberrations.

As a result of these studies a very great simplification of the methods of lens calculation has been evolved, which should prove useful in the design of optical instruments and in the further study of the theory of aberrations by relieving the investigators of these subjects of much of the tedium of the elaborate computations heretofore necessary.

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I. Spherical Aberrations

Nearly every treatise on lenses illustrates the characteristic appearance of one of the five spherical aberrations, namely distortion, but the remaining four are referred to if at all only by rather vague, indefinite descriptions. As a consequence very few have any definite conception of the distinctions between them.

All of these are shown in the accompanying figure which represents eight sections of the field as seen in a microscope or telescope, each one of these sections illustrating an instrument with a different correction or adjustment. The two sections illustrating positive and negative distortion show the two familiar effects when this aberration is under or over corrected.

Curvature is shown in the two phases dependent on the adjustment of the instrument. It is the effect so familiar to users of the microscope, because a certain amount of curvature is uncorrected in some of the best and most expensive instruments. In such a case, when the edge of the field is in focus the center shows as out of focus, lines and points becoming broad and vague, and when the fine adjustment is turned the central portion of the field becomes sharp and clear while the edge is indistinct.

Axial aberration is rarely seen because this is the distortion to which first attention is given in designing an instrument and is perhaps the easiest aberration to correct. When it is present, due to the use of an instrument under conditions for which it was not designed, it is usually associated with one of the lateral aberrations, coma or astigmatism.

Astigmatism resembles in some particulars the appearance of curvature, when the focus is made sharp on the center of the field, but differs most strikingly by the fact that radial lines remain sharp as shown at three parts of the letter S in the figure. This aberration is due to the fact that the portions of the image produced by the different zones of the lens are displaced radially and shifting of this kind would not increase the width nor therefore decrease the sharpness of a radial line. This kind of aberration is entirely different in character from the out of focus effect due to curvature.

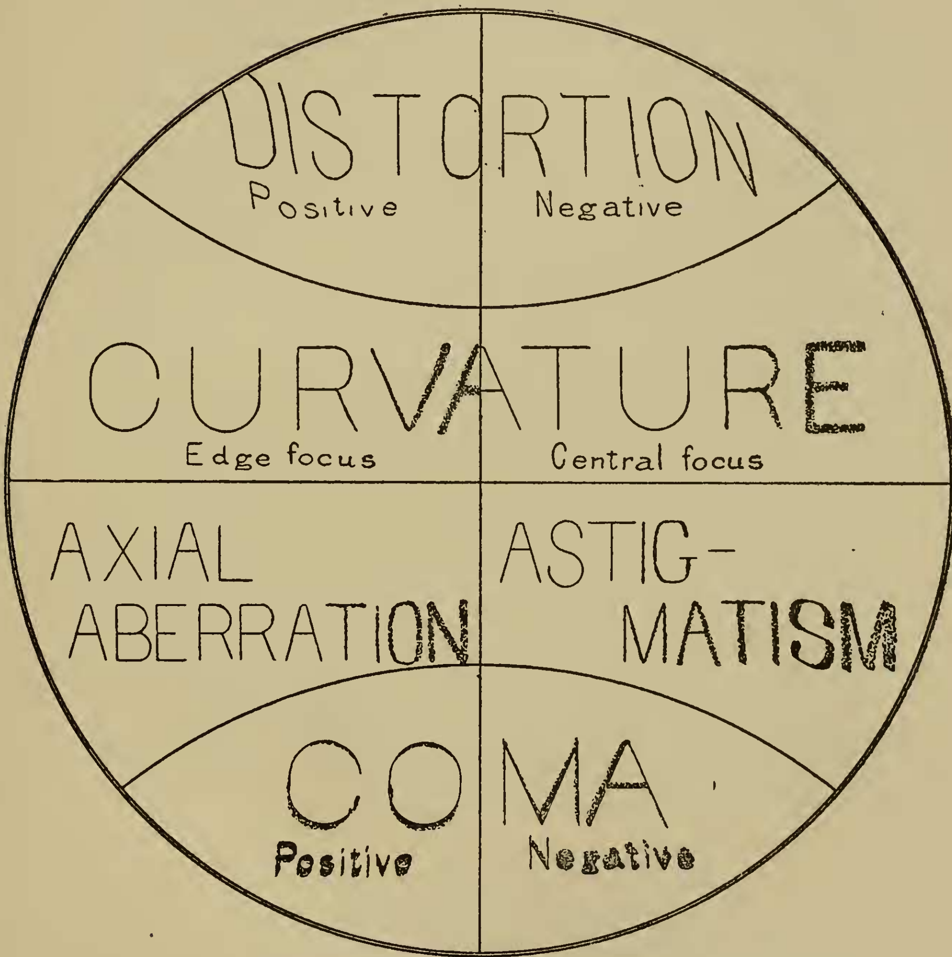

Fig. 1. Chart illustrating the appearance under the microscope of the five spherical aberrations.

Coma is also due to a radial shifting of images, but is distinguished by the fact that one edge of the image is sharp and distinct instead of fading off in both directions as is true of astigmatism. Like distortion positive and negative coma, representing under and over correction, are strikingly different in appearance.

In the calculation of aberrations, a series of rather complicated formulæ have been developed which are supposed to be measures of these aberrations, and in some cases faulty definitions of the aberration have sprung from these mathematical formulæ. Among these is a prevailing conception of the nature of astigmatism, not conforming to anything that can be verified experimentally, as has been pointed out by the writer in another place ("Science", vol. XLVII, pp. 459-460).

These effects shown in the figure are the things visible to the eye that the systems of calculation have been devised to measure in order that lenses can be so designed as to eliminate them.

As has been already intimated, it is not usual to find these aberrations present singly as here drawn, but they are all distinct enough so that they can be distinguished even when simultaneously present. Thus the distortion of an image would not prevent the recognition of any of the other aberrations that might be present.

When curvature is involved, a manipulation of the fine adjustment might be necessary to make the distinction, and between coma and astigmatism the distinction might require changes in the method of illumination.

II. Laws of Focus Formation

The study of the behavior of rays of light from a point source when refracted at a spherical surface presents no special difficulties, but the laws of focus formation under these conditions do not seem to have been formulated, though they lie at the foundation of all practical optics. The attempt made below to formulate these laws presents nothing new and nothing not thoroughly accepted by physicists.

The need of the definite statement of these laws will be evident when it is appreciated that the prevailing theory of astigmatism is contrary to these fundamental conceptions. It is perhaps one of the most remarkable facts in the whole history of optics that the hypothetical conoids of Sturm should have been accepted by physicists and used by all practical computers of optical instruments in the face of the recognized fact that nothing corresponding to them can be obtained experimentally. When it is fully realized that the theory based on this conception also violates the fundamental laws of focus formation, the study of these interesting mathematical forms will be removed from optics to their proper place in geometry and in their place rational methods of measuring the astigmatism will prevail.

The laws of focus formation as regards object points on the optical axis are correctly interpreted even in elementary treatises, but it is not so well understood that precisely the same laws apply to oblique refraction. It is not difficult to state them in such general terms that their universal applicability is at once evident.

Law I. All possible foci of an object point lie on the line through that point normal to the spherical refracting surface.

This law has often been stated in the form "the focus of a point is a line" without defining the position of the line. The law is true of all refractions of a point source of light through a single spherical surface. The equation of this line may be given in either of the following forms:

(1)
$$\frac{n'}{c}-\frac{n}{c'}=\frac{n'-n}{r}\cdot\frac{\cos\frac{\theta+\theta'}{2}}{\cos\frac{\alpha+\alpha'}{2}}$$

(2)
$$\frac{n}{c'}-\frac{n}{r}=\left(\frac{n'}{c}-\frac{n'}{r}\right)\cdot\frac{\cos\frac{\alpha'+\theta'}{2}}{\cos\frac{\alpha+\theta}{2}}$$

the angles θ and θ' in each case being measured from the normal to the surface.

The pair of aplanatic points are those in which $c=r\frac{n'}{n}$, $c'=r\frac{n}{n'}$, $\theta=\alpha'$ and $\theta'=\alpha$ in which case the ratios of cosines become equal to unity. The common aplanatic point is where c, c′, α and α' are all zero and $\theta=\theta'$. These are the only finite values of these angles which will eliminate the variable factor in the equation.

Law II. All rays from an object point to points on a spherical refracting surface equidistant from the line from the object point normal to the surface focus at a common point.

This law is the definition of a focus, the concentration of a very large number of rays to a point. A focus, according to this definition, is always a point. The line referred to in the discussion of the first law is in fact a train of foci. There should be separate words to designate these two conceptions. Perhaps the line should be called the locus of the object point, as will be done below.

Law III. The location of the foci of rays through successive zones depend on the relation of the angles θ and α or θ' and α'.

That which is called positive spherical aberration where the focus of the outer zones is nearer the center of curvature than of the rays of the inner zone is the condition where α increases more rapidly than θ.

The above laws refer to refraction at a single surface. The first law holds for all subsequent refractions through a centered optical system for object points on the optical axis. It is not true of the second or subsequent refractions of an object point away from the optical axis though the locus of that point may remain approximately linear. It is always more or less curved.

On the optical axis, while the locus remains linear and fulfills the conditions of Law I and all foci conform to Law II, they may fail to conform to Law III because the successive relative values of angles α and θ contribute to the final locations of the foci. The locus of an object point thus comes to be a very complicated thing even in the simple case of an axial object. Instead of attempting to study the subject exhaustively it is enough for practical purposes to determine the limits which for an axial point requires the calculation of at least two rays and for a point away from the axis at least three or four rays.

The usual discussion of the loci in oblique refraction illustrates the errors in not recognizing these laws of focus formation. For instance Figure *27* on p. 64 of Southall's "Geometrical Optics" illustrates the loci of the meridianal and sagittal planes as a pair of lines intersecting, at two points, a line passing through the central point of the ray bundle at the surface of refraction. A plane surface is only the special case of a spherical surface with an infinite radius. According to Law I all possible foci lie on the normal from the object point S which is represented by the line AS. The two edge rays on the sagittal plane $\overline{\mathrm{JS}}'$ & $\underline{\mathrm{J}}\ \overline{\mathrm{S}}'$ being equidistant from the normal should meet on this plane precisely as drawn according to Law II, but this point should be slightly further from the object point than the intersection of the median line BS′ with the normal, according to Law III. According to the same law the edge rays of the meridianal plane would intersect this normal or focus line as shown but at unequal distances. These two rays should also intersect the median or chief ray at different points and intersect each other at a third point. The most cursory examination of a figure showing the nature of a locus of a point produced by a refracting surface will convince one that on a meridianal plane rays on the same side of the normal invariably intersect each other at some distance before they come to their respective foci in the case of positive aberrations or beyond in the case of negative aberrations as seen in this case. These intersections are not foci but only individual crossings of rays. Foci result from the simultaneous convergence of many rays. In the case of a single refracting surface a focus is the convergence of all the rays intersecting the lens on a zone equidistant from the normal from the object point. Perhaps the failure to make the distinction between individual ray crossings and ray concentrations is accountable for the prevalent misconceptions. No one makes this mistake in reference to the foci from axial objects, and if an oblique focus were conceived of as the lateral half of an axial focus one would retain the true understanding of the nature of the focus.

Such a figure as this is thus seen to be inaccurate in all its details and to give a completely erroneous picture of the nature of oblique refraction. Apparently from a knowledge that this view of the nature of oblique refraction does not accord to experimental observations it is usual to contend that this represents only the behavior of a very narrow bundle of rays, so narrow that it cannot be studied experimentally because of defraction phenomena. This begging of the question does not avail, however, since there is no room for argument in this case for the reason that the laws here formulated are the direct consequences of the fundamental laws of refraction upon which the whole superstructure of Geometrical Optics rests, and apply with equal force to all refractions including those for narrow bundles.

Again in the case of curved surfaces the construction usually presented (See, for instance, Southall's "Geometrical Optics," pp. 49-50, fig. 15a and 15b) one can readily conceive what the nature of the surface must be to produce the conoid of Sturm. It would not be difficult to calculate nor would it be impossible to grind such a lens, though it would not by any means be a spherical lens.

As applied to a spherical lens it could be shown by calculations that every detail of the construction shown in these figures fails to conform to the laws above enunciated in the same definite way explained above in the case of refraction at a plane surface. The writer has calculated the rays and constructed models and verified them in every particular experimentally, and has proven beyond controversy that the transformation of a beam of light while passing through a locus behaves as the laws indicated and not at all according to Sturm's theory.

Here again the assumption is made that the construction illustrated in these figures applies only to very narrow bundles, an assumption as will be shown below necessary to give the theory any standing at all because all the observable facts contradict it and it has no more basis in sound theory than in the previous case.

The final consequence of any focus is that all rays concerned are completely reversed in relation to each other and in case lateral aberrations and a section of a beam of light beyond a locus assumes a different and characteristic shape.

A very simple and effective way of showing how the reversal is accomplished when a beam of light passes through an oblique locus is by pasting strips of paper on a common reading glass, leaving four equal windows, thus conforming

Fig. 2. Photographs showing the distribution of light at different planes in a beam of light passing through a focal region. The lettering is the same as in fig. 3.

to the diagram used to illustrate Sturm's theory. Supporting the lens obliquely in the sunlight and laying a piece of solio paper in the refracted beam thus making a permanent record of the distribution of the light at that plane. Repeating this at different planes will enable one to secure a very complete record of the transformation. A few of such records are shown in the accompanying figure.

To contrast the generally accepted hypothetical changes with those actually formed in the beam of light we may designate the following phases of the rectangular pencil forming the conoid by Roman numerals and of the actual pencil by letters: (Fig. 3).

I. *The initial phase* is the shape of the unaberrated image. We have chosen the four-celled grating commonly employed to illustrate this aberration with the intersections of the lines numbered so that the transformations may be readily followed.

II. *The first oblong phase* with a much more rapid decrease in the direction of the meridianal plane represented by the numbers 4, 5 and 6.

III. *The meridianal locus* at which the points 4, 5 and 6 are assumed to coincide. The shape of the pencil at this region is a line at right angles to the meridianal plane.

IV. *The second oblong phase* of the same general character as the first oblong phase but with the lines reversed so that the numbers read 6, 5 and 4.

V. *The second orthographic phase* in which the pencil has the same proportions as in the initial phase but with the same reversal of numbers seen in the previous phase.

VI. *The third oblong phase* where the numbers on the meridianal plane are further apart than those at right angles to this plane.

VII. *The sagittal locus* where the light all concentrates on the meridianal plane as a line along this plane which intersects the sagittal plane at a point at which 2, 5 and 8 are supposed to coincide.

VIII. *The fourth oblong phase* of the same shape as the third phase but with the position of the points 2, 5 and 8 inverted.

This is the final shape of the spreading pencil according to Sturm's theory.

The actual focal transformations are much more complicated as will be comprehensible from a study of the second set of diagrams.

(a) *The initial phase* precisely like that used in the preceding series of diagrams.

(b) *The first conjunction,* the plane where points 1 and 2 are nearest. There is a point in line between 2 and 5 which does coincide with 1 on a plane not far above this.

(c) *The first crossing,* the plane where 4 coincides with 5. The point 4 has been travelling along the line 4 and 5 and when it reaches that point the remainder of the line stretches out a short distance behind.

(d) *The second conjunction* being the plane showing the nearest approach of 1 and 3. This phase is probably in general shape mose nearly comparable with phase II, according to the Sturm theory.

(e) *The second crossing,* in which plane points 4 and 6 coincide. This phase probably could be chosen as the nearest approximation to the third phase of the old theory.

(f) *The first marginal focus* at which the points 1 and 7 coincide. It is only the beginning of the coincidence of the line 1, 4, 7.

(g) *The third conjunction* at which points 2 and 3 are nearest. This diagram also represents the completion of the reversal of line 1, 4, 7.

In this and the following diagram the constriction between the erect and inverted portion of the image is by no means as sharp as here shown where the lines indicate the edges only of the infolded image, and the whole image at the point of crossing has a very appreciable width, still the general shape of the pencil section conforms very closely with these diagrams.

(h) *The third crossing* where points 5 and 6 coincide but it is not the conclusion of the reversal of the line, because the points on the line 5, 6 are spread out behind this point. This and the preceding five phases include the region in which the reversal of the rays is brought about as is supposed to occur in the meridianal locus.

(i) *The median focus* being the coincidence of points 2 and 8. This point is probably the nearest to the sagittal locus of Sturm's system, and here also the shape of the pencil most nearly suggests the fourth oblong phase.

(j) *The optical focus* where the reversal is complete except for the last line 3, 6, 9. This is approximately the plane which will be chosen by the eye as the focus.

(k) *The second marginal focus* at which 3 and 9 coincide. This and the preceding five phases intersect the locus obliquely.

(l) *The final phase,* showing the complete transformation and approximate form of the pencil.

A striking and significant difference between these two sets of diagrams is that in the first rays 3 and 9 intersect nearest the set of refracting points and in the other diagram this intersection is furthest from that surface. The further fact that the loci of which these intersections are one limit, are nearly at right angles with one another emphasizes the irreconcilable differences between these conceptions.

The oblique locus in the case of a reading glass is not a line but a very narrow linear figure, appreciably curved at the ends, oblique relative to the optical axis and approximately normal to an equivalent single lens surface.

These diagrams exhibit only two of the five aberrations. *Distortion* and *curvature* are concerned with differences between the foci of the different portions of the field. *Axial aberration* is that which appears when the lens is normal to the beam of light. The aberrations here exhibited are *astigmatism* and *coma*. Considering point 5 the middle of the light pencil then, the 1, 4, 7 region represents astigmatism and the 3, 6, 9 region represents coma. The optical focus is on the plane at which the phenomenon known as coma is most pronounced, which occurs where there is the greatest difference between the numerical values of these two aberrations.

While there is just enough similarity between these two sets of diagrams to explain how the geometrically simpler conception of Sturm was suggested and enough correspondence of the focal lines of that theory with the mean of the focal values when correctly determined to make the calculations of some practical value, the whole theory should certainly be replaced by one conforming to physical observations.

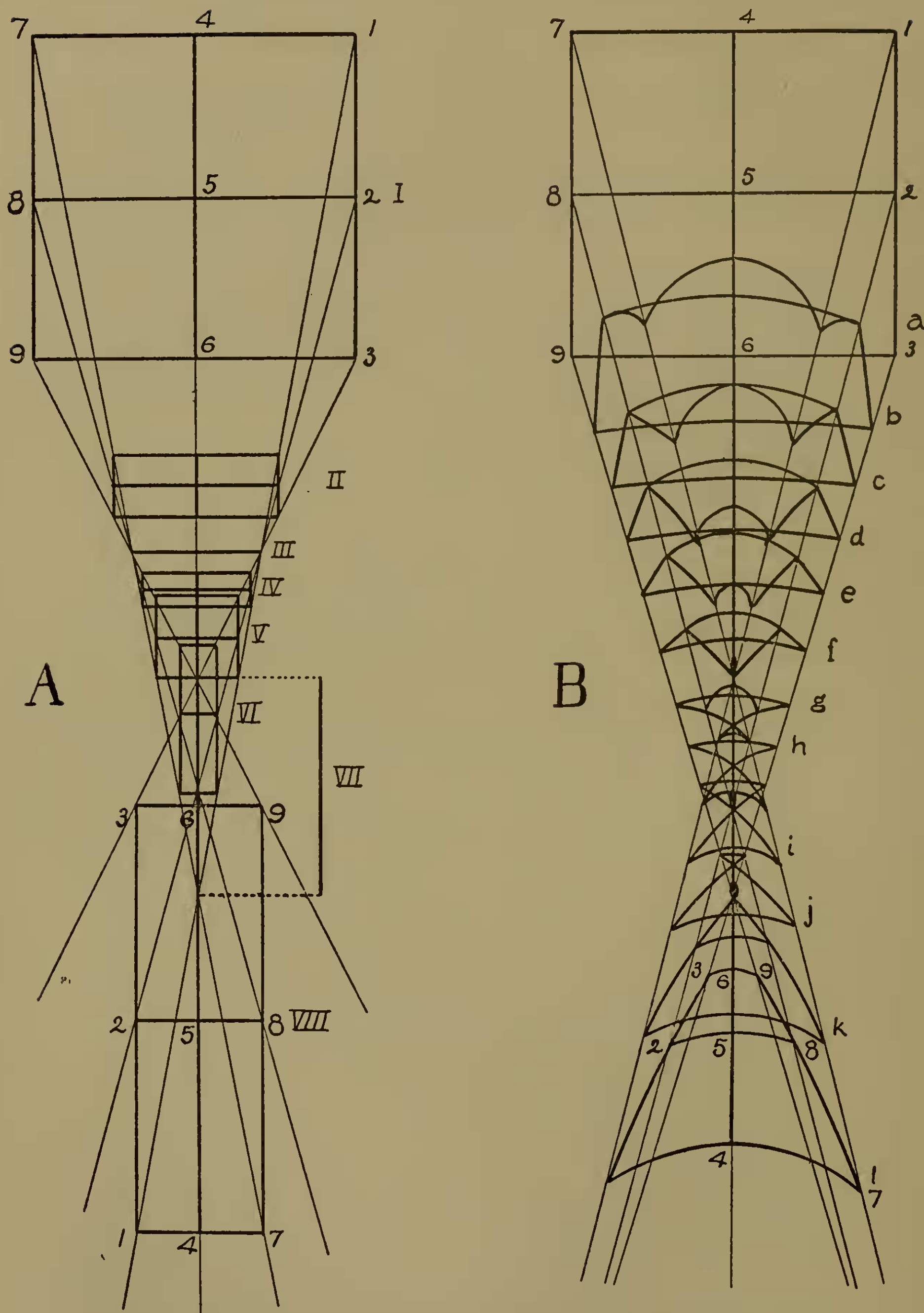

Fig. 3. Diagrams contrasting the inversions of the rays in a beam of light rays on passing through their foci. (A) According to Sturm's theory, (B) according to observed changes (See fig. 2), (I–VIII) successive hypothetical phases, (a-1) successive actual phases, (1–9 rays) whose positions are traced through each plane.

The differences just pointed out have not been wholly unknown to physicists by any means, but apparently it has not been made clear that the differences are due to fundamental laws of focus formation equally applicable to narrow as well as wide bundles of rays.

III. Oblique Axis Calculations

The system of lens calculation proposed by the writer consists of the calculation of a definite series of rays through an optical system, either graphically or mathematically, and in the latter case the plotting of these rays on an enlarged drawing of the lenses. When this is done a simple inspection of the drawing will enable one to estimate the simultaneous effect upon the aberrations of any possible variation in the construction data.

The rays that need be calculated are as follows:

1. The *Zone* ray, commonly called the edge ray, the extreme ray from the point of intersection of the optical axis and the object. This ray determines the *aperture* of the instrument.

2. The *Field* ray from the extreme edge of the field midway between the extreme rays of the pencil. This ray determines the *magnification* of the instrument.

3. The *Paraxial* ray from the object along the optical axis. The difference between this and the Zone ray, measured on the X axis, determines the *axial aberration,* that between this and the field ray also measured on the X axis determines the *curvature.*

4. The *Distortion* ray similar to the Field ray but from a point midway between the center and the edge of the field. The difference between this ray and half the Field ray, measured on the Y axis, determines the *distortion.*

5. The *Comatic* ray, similar to the Field ray but through the nearest marginal point of the lens system. The difference between this and the Field ray measured on the Y axis or with the Zone ray measured on the X axis determines the *coma.*

6. The *Astigmatic* ray, similar to the Comatic ray, but through the most distant marginal point of the lens system.

Measured the same way as the Comatic ray. This ray determines *astigmatism.*

The focus of the Zone and Paraxial rays is determined by the intersection with the optical axis. The Distortion ray does not require the determination of a focal point. The three oblique rays require, for the three values of the X coordinates, the determination of the oblique focus of each, which can be most easily accomplished by the new oblique axis method described below. There is no true focus in the strict sense of the word after a number of refractions, but if each successive focus were considered the source of a radiant pencil of light then all would be true foci. The theoretical foci secured by assuming this character of each focus lie within the loci of the point and this is the best if not the only method available for determining the position of an element of the locus of a point away from the optical axis after numerous refractions.

The oblique axis method is based on the first law of focus formation, that all possible foci lie on a line from the object normal to the refracting surface. Having calculated the path of a ray in any of the usual ways, the determination of the focus after the first refraction is accomplished according to this method by locating the normal on which the focus must lie and finding the intersection this normal makes with the refracted ray which is accomplished by running a line from the object point towards or through the center of curvature. After thus locating the focus on the ray path after the first refraction, this point is considered as an object and the focus conjugate with it after the second refraction is determined in the same manner.

The accompanying figure (4) illustrates the calculation of the foci of a ray in both directions through a doublet, assuming the object to be first on one side of the lens and then on the other at the two points marked Focus 0. The graphic calculation of these rays is shown above.

The foci 0 and 1 are on Axis 1 normal to the first surface, foci 1 and 2 are on axis 2 normal to the second surface and foci 2 and 3 are on axis 3 normal to the third surface.

The application of this method to the study of aberrations is seen in figure 5 where the successive foci of two points

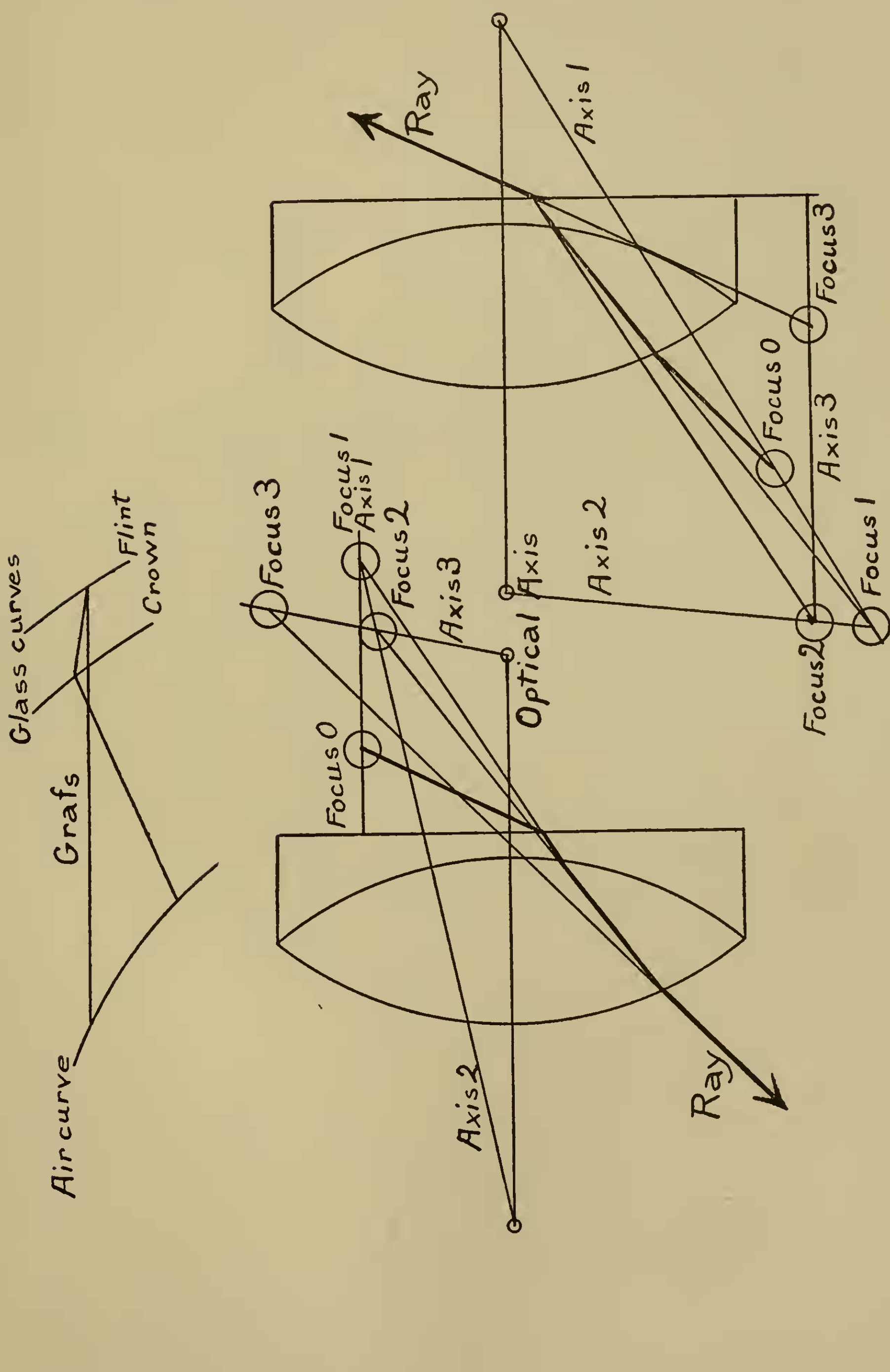

Fig. 4. Graphic calculation of consecutive foci through a doublet.

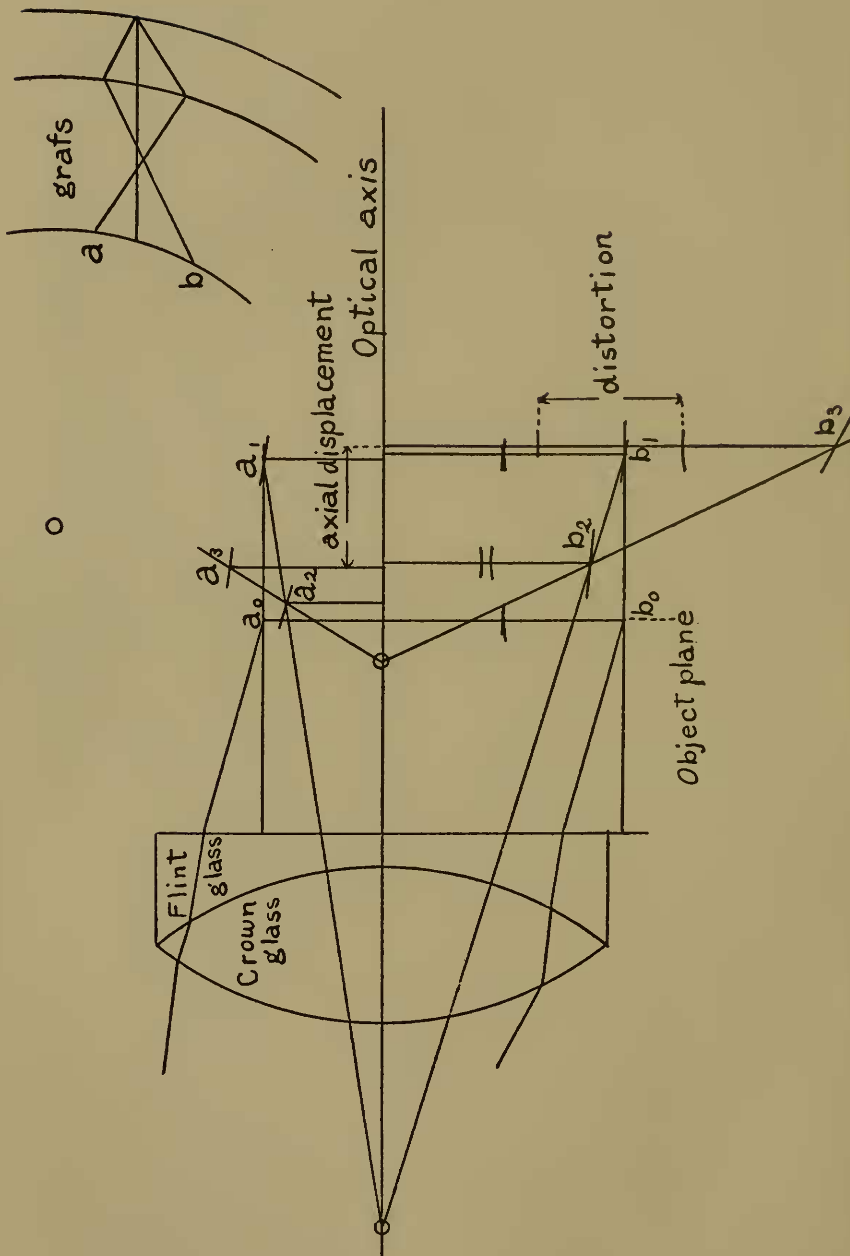

Fig. 5. Graphic calculation of two rays and successive foci to illustrate distortion and lateral aberration.

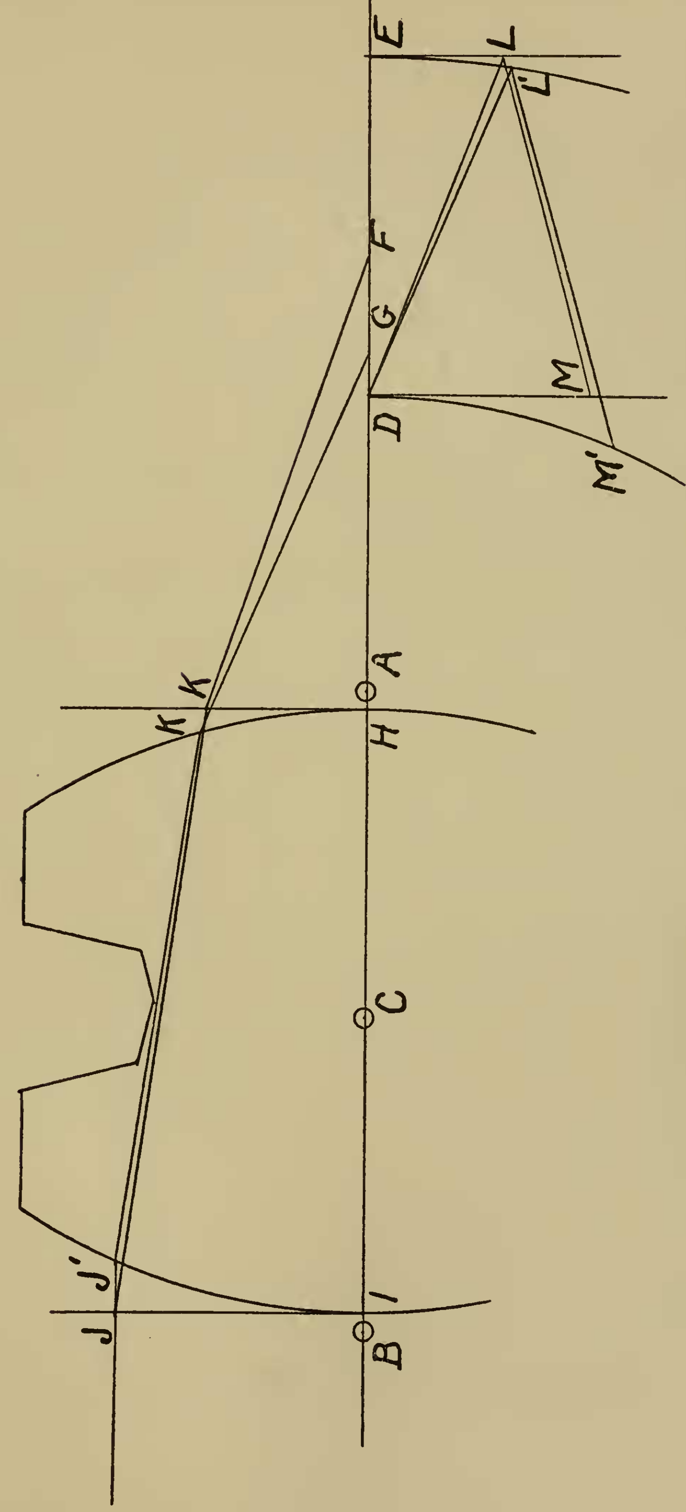

Fig. 6. Graphic calculation of edge and paraxial rays through a Coddington lens showing axial spherical aberration.

a and b through a doublet objective is studied. One of these points is twice as far from the optical axis as the other. Parallel rays from these two points to the lens are selected to show at the same time distortion and oblique aberrations. The graphic calculation of the two rays is given to the right above and all the oblique axes are drawn, the successive foci being represented by letters with subnumbers. Each image plane is also drawn and on the longer the length of the shorter is laid off by short curves. The distance between these curves is the amount of the distortion which is not serious till at the third refraction. Because of the lateral aberrations, the axial displacement is quite large at the second refraction and very serious at the third.

A simple inspection will show that both axial displacement and distortion will be most greatly improved by a slight increase in the length of the radius of the third surface, and that this would have a very much more profound effect than a change in the curvature of the second surface, and that if these two are changed in the proper degrees the aberration could be greatly improved without changing the magnification of the lens as a whole.

The great advantage of this system of lens calculation is the facility with which the manner of correcting aberrations can be located by inspection of the drawings.

The graphic calculation, which is preliminary to the oblique axis calculation in these examples, is described below.

The method of calculation is illustrated in Figure 6, which shows the calculation of the axial spherical aberration at the principal focus of a Coddington lens. A is the center of curvature of the first surface, B of the second. C is the Graph Center, D and E are so located that CD/CE=n/n′.

The method of calculation of a ray parallel with the optical axis is as follows:

1. Draw DL′ parallel with J′A
2. Draw J′K′ parallel with CL′
3. Draw L′M′ parallel with K′B
4. Draw K′G parallel with CM′

The point G is the principal focus for the zone J′ of the lens.

Exactly the same method applies to the calculation of paraxial rays except that straight lines perpendicular with the axis replace all the curves. The steps in the process are the same as those given above, omitting the primes and ending at F, which is the principal focus for paraxial rays. The distance FG is therefore the spherical aberration.

The correctness of this method of calculation is proven as follows: In the triangle CDL′, L′DE is by construction $= \alpha$ and since CD/CL′ = n/n′ the angle DL′C $= \alpha'$. Since θ equals zero and the external angle L′DE = CL′D + DCL′ we have from the equation $\alpha - \theta = \alpha' - \theta'$; DCL $= \theta'$ and therefore CL is parallel with J′K′. In the same way, in the next refraction CM′ can be shown to be parallel with K′G.

In the calculation of the paraxial ray it is at once evident that a ray half way between I and J would by this method of construction proceed to a point half way between H and K and then exactly to F. The same would be true of a ray ¼ or ⅛ above I and H, that is the distance from the optical axis does not affect the focal distance F; therefore a paraxial ray comes to a focus at this point.

The methods above described constitute a complete scheme for lens calculation which can be carried out grafically with as much accuracy as is required for practical purposes, since it is well within the limits of the accuracy of the physical data and mathematically to any degree required for theoretical investigation.

IV. On the Aberration of Depth

The aberrations due to the thickness or depth of objects have received very scant attention though it is well known through observation, as well as from the theoretical considerations, that there are such aberrations.

The figure accompanying this article (Fig. 7) illustrates the amount and character of such aberrations in the simple case of a single lens surface, and of an object limited to a single axial plane.

It illustrates at the same time an application of the methods of graphic calculation and the new oblique axis method of calculation, which is available either for graphic or mathematical computations. The "air curve" and "glass curve" are drawn in the manner already described about the "graph center" with radii proportional to the index of air and of glass, respectively, and the graphs, three series of which are shown, are drawn parallel with three radii of the lens from the three points of refraction being studied.

The object whose images are studied is the letter A standing edgewise to the lens and at some distance above the optical axis. The "chief image" is the one produced by the central portion of the lens and the incident pencil is indicated by lines from the principal points of the image to the middle point of the lens.

Lines from the graph center to the air curve parallel with three lines of the incident pencil, but which are omitted in the drawing, locate the air ends of the chief graphs. Similar lines, also not shown, from the graph center to the glass ends of these graphs give the directions of the corresponding rays of the refracted pencil.

In precisely the same manner the pencils to the extreme edges of the lens which produce images showing extreme aberrations are indicated by lines, and likewise parallels to the rays of these pencils from the graph center to the air curve locate the positions of the other graphs shown. Likewise also the directions of the rays of the refracted pencil are indicated by lines connecting the glass ends of these graphs with the graph center. None of these lines from the graph center are drawn nor need they be drawn when making graphic calculations since the graphs themselves are the only record that need be made of the process of calculation.

The "oblique axis" method of calculation is as stated above based on the fundamental fact of refraction at spherical surfaces that the focus conjugate with any image point must lie on a line through that point and the center of curvature of the lens. Thus all points on the optical axis have their conjugate foci also on the optical axis. In the same manner all conjugate foci away from the optical axis lie on oblique axes. The oblique axes of the principal points of the object A are shown in the figure. These axes are drawn away from the lens because the object A is so located that virtual images will be formed in that direction.

Knowing the direction of each ray of the refracted chief pencil as explained above, it is only necessary to find the intersection of a line from the vertex of the lens, which is